RECENT PARALLELS BETWEEN THE PHILOSOPHY OF SCIENCE AND MATHEMATICS

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Following World War I European philosophy of science formed an alliance with mathematics culminating in an attitude of certainty and autonomy that rejected all nonempirical claims to truth and purported to make all science presuppositionless. The rise and fall of logical positivism has been one of the major themes of twentieth century thought and illustrates the danger of placing too much emphasis on science and knowledge. The restriction of rational inquiry to the modes of scientific verification and the processes of mathematical logic was far too confining for the containment of truth. Even after the rejection of logical positivism as a general epistemology on the ground that not all empirical knowledge was like scientific knowledge, it continued to survive as a philosophy of science and became the dominant influence on American science after World War II. Thus science became isolated from other realms of thought. This dichotomy has only recently been bridged by a more historically oriented approach. The philosophies of science and mathematics have experienced a number of parallels in the development of these trends.

In the nineteenth century natural science was accepted as a free, autonomous, reliable means for the steady accumulation of truth. Beginning with the facts of observation and using only tested modes of inference, science moved carefully to higher levels of generalization. In an effort to analyze the success of this procedure an early form of positivism was developed by Ernst Mach in which science was viewed as a growing body of concepts which can be directly connected with facts given by direct sensation.[1] Excluding any a-priori elements such as absolute space and time, all scientific statements had to be empirically verifiable and thus capable of reduction to statements about sensations. The difficulty with this approach was that scientific principles contain mathematical relationships not reducible to sensations alone. Furthermore, the development of formalism in mathematics by David Hilbert had shown that mathematical structure can be separated from meaning yielding a purely formal system. Thus no mathematical system of axioms and deductions by itself says anything about the world. Although this insight freed mathematics from science, it raised the fundamental problem of how classical Newtonian theory can form an axiomatic system and at the same time describe the actual world of nature in contrast to all other logical possibilities. One solution to this problem was the conventionalism of Poincare which viewed scientific laws as mere conventions about facts of science. The theoretical terms of a scientific law could presumably be replaced by phenomenal language so that mathematical relationships are only conventions for expressing relations between phenomena.

Mach's positivism strongly influenced Einstein in the development of relativity theory[2] but the gradual acceptance of the new physics with its growing emphasis on mathematical theory put increasing strain on the philosophy of science. Conventionalism helped to end the dependence of physics on Euclidean geometry, but left science without any basis of agreement at the theoretical level. The view of Poincare and Hilbert that axioms were only definitions seemed to be too arbitrary a foundation for science or mathematics. Hilbert's formalism had been preoccupied with the structure and consistency of mathematics and had set aside the question of the meaning of mathematical concepts. The logical school initiated by Frege remedied this omission with its concern to analyze mathematical concepts. Frege expressed the problem as follows. "Strictly speaking, it is really scandalous that science has not yet clarified the nature of number. It might be excusable that there is still no generally accepted definition of number, if at least there were general agreement on the matter itself. However, science has not even decided on whether number is an assemblage of things, or a figure drawn on the blackboard by the hand of man; whether it is something psychical, about whose generation psychology must give information, or whether it is a logical structure; whether it is created and can vanish, or whether it is eternal."[3] Frege held that numbers are logical objects which exist independent of human definitions, and thus that the laws of number could be reduced to logic. Whitehead and Russell extended this logistic thesis by their monumental effort to reduce all of mathematics to logic in the <u>Principia Mathematica</u>.

Logical positivism was the product of a group of scientists, philosophers and mathematicians, known as the Vienna Circle, who were strongly influenced by the sensationalism of Mach and the logicism of the <u>Principia Mathematica</u>. They concluded that if mathematics could be reduced to logic then mathematical statements of scientific laws could also be given in terms of mathematical logic. Thus a scientific theory could be axiomatized by logical relations between theoretical terms if a way could be found to link these theoretical terms with the observation terms used to describe phenomena. This was a problem that Wittgenstein had developed in his <u>Tractatus</u>[4] which was an attempt to show how the atomic propositions and associated logic of the <u>Principia Mathematica</u> could serve as a theory of language. He showed that if it were assumed that atomic facts correspond to atomic propositions, and if all other propositions except declarative ones could be eliminated, then a logically perfect language and a literal description of the world would be possible.

The Vienna Circle changed Wittgenstein's assumptions into assertions by interpreting atomic propositions as reports of sensory observations. They linked the elements of the idealized language with facts about the world by correspondence rules to ensure the meaningfulness and truth of each proposition. In their opinion the cumulative success of science consisted in adding to the growing number of such propositions, and linking them to obtain more complex propositions. This process appeared to offer a way to reform philosophy and integrate it with physics and biology into a single unified science. At the same time metaphysical entities could be avoided since they involve theoretical terms that cannot be linked to explicit observational definitions. According to this verificationist theory of meaning all cognitively significant discourse about the world must be empirically verifiable. The ambitious hopes and confident certainties of the logical positivists came under attack almost from their inception. Part of this confidence was based on the success of classical physics. The challenge of the new physics culminated in the development of quantum theory and the replacement of many classical ideas and assumptions. Heisenberg' s uncertainty principle revealed inherent experimental and conceptual limitations in science. Precise values of position and velocity in an atomic system cannot be simultaneously determined. The wave-particle dualism of atomic physics demonstrated that scientific theories are not literal descriptions of nature, but require different models in describing different experimental situations. Moreover the process of observation influences what is observed and thus concepts inevitably enter descriptions of atomic events. The positivist response to modern physics was to urge that all models be discarded and to be content with equations that correlate observables. But most physicists preferred to retain both wave and particle models while recognizing their limitations. Mary Hesse argues that models function in science as more than literal descriptions of nature, but as devices which "can be generalized, extended and tested, and if necessary modified, as a purely formal deductive system cannot." [5]

Another goal of logical positivism, the reduction of all knowledge to a single unified science, was thwarted by developments in mathematical logic undertaken by one of the members of the Vienna Circle. Using a meta-mathematical method introduced by Hilbert, it was shown by Gödel that axiomatic systems such as Principia Mathematica, if consistent, must necessarily be incomplete. Furthermore, in any formal system that includes the theory of natural numbers there are undecidable arithmetical propositions, and arithmetical concepts can be found which are not definable within the system. Carl Boyer in his history of mathematics says of Gödel's proof, "it appears to foredoom hope of mathematical certitude through the use of obvious methods. Perhaps doomed also, as a result, is the ideal of science - to devise a set of axioms from which all phenomena of the natural world can be deduced."[6] Even more disconcerting is a theorem proved by Skolem which says that it is impossible to characterize the number series by a finite number of axioms.[7] This is not the kind of logical foundation that will secure a unified axiomatic structure for all of science.

Further attempts at establishing the autonomy of science were equally unsuccessful. The verifiability criterion was found to be overly restrictive since it excluded all universal statements such as "all sodium samples react with chlorine." Many scientific laws are universal statements, but no finite set of observations can establish these statements as true. Falsifiability also proved to be too restrictive as a condition of empirical meaningfulness since it excluded existential statements such as "there exists at least one binary system of white dwarf stars." Such empirically significant statements cannot be falsified by any finite set of negative observations. An alternative approach introduced by Rudolf Carnap was an attempt to construct an "empiricist language" into which scientific laws, but not metaphysical statements, could be translated.[8] Although he was unable to define dispositional properties like temperature and electric field strength using explicit observation terms, he was able to introduce them into the empiricist language by means of "reduction sentences" and "operational definitions." But since operational definitions leave concepts undefined for cases in which the operation is not performed, they specify only a partial meaning and thus have an "open texture" subject to further specification in terms of new operational procedures. Even these modified criteria proved to be too restrictive for such concepts as the wave function in quantum mechanics. Theoretical terms of this type are not definable in the observational language but are part of an axiomatic system linked in its entirety to operationally-defined concepts. Thus, it has been conceded that empirical significance applies in general only to the entire system of sentences which constitute a given theory.[9]

Meanwhile, Wittgenstein had dissociated himself from the logical positivists and had become an increasingly skeptical onlooker. In the *Tractatus* he had shown how a formalized theory in science could provide a representation of facts about nature. But he recognized the problem of applying any axiomatic formalism to the world as we know it. The positivists believed that all the abstract terms of a meaningful theory can be associated with sensations or observations. But Wittgenstein argued that an axiomatic theory defines only logical possibilities which cannot be brought into complete correspondence with the natural world because logical relations hold only within a symbolism or language. Even in the *Tractatus* he recognized this problem: "So too the fact that it can be described by Newtonian mechanics asserts nothing about the world; but *this* asserts something, namely, that it can be described in that particular way in which as a matter of fact it is described. The fact, too, that it can be described more simply by one system of mechanics than by another says something about the wor1d."[10] The connections between 1anguage and reality are determined by usage rather than logic; thus the relationships between words and the world cannot be given by formal definitions.

In Wittgenstein's later writings he emphasized that unconceptua1ized sensations cannot play a direct part in building a language that will ensure autonomy and certainty. He agreed with Kant that "Percepts without concepts are blind," and went on to find alternative ways of showing how language operates. He suggested that our understanding of conceptual relationships should be based on a wider study of how language uses are estab1ished both individually and culturally in order to determine what can meaningfully be said. This suggestion was followed up in mathematics by Friedrich Waismann's *Introduction to Mathematica1 Thinking* which provided a conceptual analysis of the word "number." It showed how many philosophical problems can be avoided if, for example, questions that have meaning for only integers are not asked about irrational numbers, or concepts that are used in the development of real numbers are not applied unthinkingly to complex or transfinite numbers. The posthumous publication of Wittgenstein's *Philosophical Investigations* in 1953 made it clear that the philosophy of science must take account of the fine texture and diversity of language usage, and the dangers of assuming an unproblematic correspondence between language and reality.

One of Wittengenstein's students, N.R. Hanson, began to apply this lesson to science by a process of historical analysis of scientific laws.[11] Hanson found that Newton's laws of motion had been put to a wide variety of uses. For example, the second law was used as a formula to calculate the acceleration of various systems, as an empirical generalization, as a definition of force, and as a rule for measuring forces. In each of these diverse uses the meaning of the law depends on a system of statements that show how the law is used. He argues that observation reports are shaped in these different uses by the conceptual scheme of the observer and thus are "theory-laden." Since there is no theoretically neutral observation language, the goal of a foundationalist-inductivist account independent of theoretical concepts is doomed to failure. But this was just the beginning of the historical-conceptual attack on autonomous science.

Publication of Thomas Kuhn's *The Structure of Scientific Revolutions*[12] in 1962 marked the next stage of the revolt against the idealizations of logical empiricism. Kuhn distinguished between "normal" science during periods, when a paradigm is widely accepted as a model or ideal of explanation, and periods of revolution when there is no agreement between competing paradigms. Such "paradigms" determine the kinds of problems scientists are interested in, as well as their standards for solving those problems and the very nature of the facts which are considered relevant. A "revolution" occurs in a shift from one paradigm to another, which does not involve purely logical steps, but is at least partly a matter of personal or group commitment to a new pattern or perspective. The study of hisorica1 revolutions revealed science as it actually is rather than as it should be, showing its dependence on psychological and sociological as well as logical factors. It also demonstrated the differences between scientific theories and interpreted axiomatic systems, and how the philosophy of science was affected by ignoring these differences.

Extending Kuhn's historical approach, Stephen Toulmin rejected the distinction between normal and revolutionary science with an argument for the continual evolution of concepts.[13] Because of these constantly shifting concepts in the development of science there can be no foundational propositions at all. Kuhn's normal science could yield to logicalist analysis, but for Toulmin there is no conceptual stability available for the application of logical criteria. The context of discovery in the process of theory-changes cannot be isolated from the context of justification, which had been the basis for the logical analysis of scientific theories.

An even more radical departure is represented by Paul Feyerabend who urged that no single theory should ever by allowed to stand by itself as the ideal of normal science.[14] Since no one theory can ever account for all of the facts, competing alternatives must be permitted so that all possibilities can be tested. Furthermore, there is no neutral set of observation statements by which mutually incompatible theories can be logically compared. The adoption of a particular theory alters the meaning of the facts to be accounted for, so that competing theories are incommensurable with one another. The progressive view of science that required new theories to be consistent with earlier theories in the same domain is not possible, precluding any kind of cumulative inductive logic. Feyerabend's historical studies led him to conclude that theories are established by propaganda, persuasion and ideology. Order is subjectively imposed on selected data rather than objectively determined by empirical and logical methods.

Some parallels to Feyerabend's subjectivism can be seen in the development of the intuitionist school of mathematics associated with the Dutchman Brouwer. According to the intuitionists mathematics is a free activity of thought, a creation of the human mind.[15] In this view mathematics does not need the support of an extended logic nor of a rigorous formalization. Instead of following the logicists and formalists in their attempts to build a secure foundation for higher levels of mathematical work, the intuitionists try to build mathematics at all levels by the methods of intuitive construction. They see mathematics not as one system, but as a multitude of systems each developing the inferences of arbitrarily chosen assumptions. Stephen Barker criticizes this view as follows: “The strangeness of the doctrine is thrown sharply into relief when we realize that it is a consequence of Kant's theory, and presumably of Brouwer's, that the laws of number hold true only of things as the mind intuits (senses) them, not of things as they are in themselves. The view that number is inapplicable to things as they really are in themselves means that things in reality are neither one nor many. This is too close to self contradiction to be plausible."[16] A more moderate position is suggested in Wittgenstein's view that mathematical propositions are true if they are provable in a calculus. The mathematician is an inventor rather than a discoverer, but mathematics is more than a game because numbers have uses outside of mathematics which give it meaning.[17]

These subjectively oriented approaches in the philosophy of science and mathematics place a healthy emphasis on creativity and diversity, but they seem to replace foundationalist autonomy with relativism and free-spirited anarchy. What seems to be needed is some alternative between these extremes that can retain at least some measure of objectivity and rationality for science without renewing its isolation or totalitarian claims. A balanced approach to science must include both logical inference and rational interpretation. The relations between evidence and hypothesis must be guided by logical criteria of procedure. However science cannot be limited to the coercive rules of deductive logic. Inductive generalizations are also frequently required; especially in experimental work, these involve many ingenious logical techniques. A third type of inference, often used at the theoretical level, is the hypothetico-deductive or retroductive argument which seeks to find explanatory hypotheses that will make the given data more intelligible through their concepts and deductive consequences. Since this broader approach to logicality in science is not restricted to deduction, it allows for the kind of tentativeness and disagreement that is apparent in the history of science without abandoning objective methods and logical criteria.

But in addition to these more formal types of inference, science also involves an interpretive dimension which is dependent on a wider context of meaning and structure. Although interpretation is often controversial, it is a part of scientific rationality and cannot be reduced to the kind of logicality that the logical positivists sought in their attempt to isolate science as a noncontroversial autonomous realm of knowledge distinct from other types of human activity. The category of rationality is especially applicable to the conceptual shifts, and even larger revolutions that occur in science, since these often exhibit the highest forms of human rationality. Interpretation enters into every level of scientific activity, beginning with the perception and selection of data which are shaped by our conceptual system, and extending to the norms of simplicity and fertility that govern inductive and retroductive inferences. The subjective aspects of interpretation do not necessarily eliminate the constraints and corrections of the objective order. Interpretive skills must be learned and depend on the consensus of shared experience developed within a group and sensitivity to related disciplines. Interpretation is guided by such objective norms as correct prediction and more effective control of nature. Rationality is not independent from logicality, but often utilizes the modes of logical inference in the process of conceptual selection and interpretation.[18]

A number of lessons are apparent from these recent parallels in the development of the philosophy of science and mathematics. Scientific certainty appears to be an unattainable goal, and any literal description of nature is a naive hope that must give way to the use of theoretical concepts and models. Scientific knowledge does not progress in a steady cumulative fashion, and cannot be unified into a complete and consistent axiomatic system. Empirical significance and meaning cannot be limited to observational language, but depend on an entire conceptual system including its logical relationships and language uses.

Historical analysis has shown that science is further dependent on psychological and sociological factors in its choice of paradigms, and that scientific concepts are constantly changing in the process of theory development and competition. Science is thus seen as a creative human activity requiring a wide variety of logical techniques and interpretive skills. It cannot be reduced to logicalist or foundationalist autonomy, but is dependent on a wider context of meaning and ideas.

These lessons offer new possibilities for the coexistence and even mutual concern of science and Christian faith. The inability of science to attain certainty reflects the Christian view of man's finiteness and fallen nature. The failure of logical positivism to establish a unified axiomatic foundation leading to a logically necessary order in nature leaves open the Christian idea of a contingent creation controlled by God in a logically consistent order. The broader criterion of meaning required by empiricism in the use of coherent conceptual systems correlates with metaphysical and theological approaches previously rejected as nonsense by the earlier inadequate verificationist criterion requiring complete correspondence with perceptual sensations. The dependence of science on historical antecedents and communal commitments parallels the Christian experience of a continuing community of faith. The elements of creativity in science and mathematics reveal to Christian faith the uniqueness of man created in the image of his Creator and given dominion over an objectively ordered creation.

In view of these considerations science can again take its proper place as a cultural activity sharing many of the limitations and uncertainties of other human endeavors. Science can no longer maintain its claim as the ideal for human knowledge and perfection. Stanley Jaki summarizes these conclusions in the following words: "Thus for the Christian the ideal of perfection is tied to the ideal of the perfect man in Christ. That is, a man who searches not for narrow logic but for understanding in its broadest sense which gives justice to the facts of nature as well as to the facts of history, and which satisfies man's senses as well, as his innermost aspirations."[19] In B. F. Skinner's book, Beyond Freedom and Dignity,[20] C.S. Lewis is criticized for his suggestion that too much emphasis on science and man's power over nature leads to dehumanization and ultimately to the "abolition of man."[21] Skinner argues that the abolition of autonomous man is long overdue. In his concern to subject man to scientific control he fails to recognize two decades of scholarship in the history and philosophy of science that have resulted in the abolition of autonomous science.

Footnotes

1. Ernst Mach, The Analysis of Sensations (New York: Dover Publications, 1959).

2. Gerald Holton, Thematic Origins of Scientific Thought, Ch. 8 (Cambridge, Mass: Harvard University Press, 1973).

3. Quoted in Friedrich Waismann, Introduction to Mathematical Thinking, p. 107 (New York: Harper and Brothers, 1959).

4. L. Wittgenstein, Tractatus Logico - Phi1osophicus (London: Routledge and Kegan Paul, 1922).

5. Mary Hesse, Forces and Fields, p. 22 (London: Nelson, 1961).

6. Carl Boyer, A History of Mathematics, p. 656 (New York: [-- II John Wiley, 1968).

7. See Friedrich Waismann, op. cit., p. 105.

8. Rudolf Carnap, “The Elimination of Metaphysics through. Logical Analysis of Language", trans. by A. Pap (1932), in Logical Positivism, ed. by A.J. Ayer, pp. 60-81 (Glencoe: Free Press, 1959).

9. See Carl Hempel, "Problems and Changes in the Empiricists Criterion of Meaning" in Revue International de Philosophie, Vol. 4, 1950, reprinted in Meaning and Knowledge, ed. by E. Nagel and R. Brandt, pp. 17-27 (New York: Harcourt, Brace and World, 1965).

10. L. Wittgenstein, op. cit., proposition 6.342.

11. N.R. Hanson, Patterns of Discovery (Cambridge: Cambridge University Press, 1958).

12. Thomas Kuhn, The Structure of Scientific Revolutions (Chicago: University of Chicago Press, 1962).

13. Stephen Tou1min, "Does the Distinction between Normal and Revolutionary Science Hold Water?” in Criticism and the Growth of Know1edge, ed. by I. Lakatos and A. Musgrave, pp. 25-47 (London: Cambridge University Press, 1970).

14. Ibid., Paul Feyerabend, "Consolation s for the Specia1ist”, pp. 197-230.

15. See Arend Heyting, "The Intuitionist Foundations of Mathematics,” in Philosophy of Mathematics, ed. by P. Benacerraf and H. Putnam, pp. 42-49 (New Jersey: Prentice-Hall, 1964).

16. Stephen Barker, Philosophy of Mathematics, p. 77 (Englewood Cliffs, N.J.: Prentice-Hall Inc., 1964).

17. See R.L. Goodstein, "Wittgenstein's Philosophy of Mathematics," in Ludwig Wittgenstein: Phi1osophy and Language, ed. by A. Ambrose and M. Lazerowitz, pp. 271-286 (New York: Humanities Press, 1972).

18. See Ernan McMullin, "Logicality and Rationality: A Comment on Tou1min' s Theory of Science," Boston Studies in the Philosophy of Science, Vol. 11, ed. by R-. Cohen and M. Wartofsky, pp. 209-224. (Dordrecht: Reidel Publishing Co., 1974).

19. Stanley L. Jaki, Science and Creation, p. 158 (New York: Science History Pub1ications, 1974).

20. B.F. Skinner, Beyond Freedom and Dignity, p. 191 (New York: Bantam Books, 1971).

21. C.S. Lewis, The Abolition of Man (New York: The Macmillan Company, 1947).